# Interaction between viscous varying modified cosmic Chaplygin gas and Tachyonic fluid

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June 5, 2013

**Abstract**

In this paper we study the interaction between the general form of viscous varying modified cosmic Chaplygin gas and the Tachyon fluid in the framework of Einstein gravity. We want to reconstruct the Tachyon potential and total equation of state parameter graphically by using numerical methods. In the presence of deceleration parameter, the interaction between components becomes sign changeable to explain different evolutionary eras in the universe. We review the potential and total equation of state parameter in Emergent, Intermediate and Logamediate scenarios of scale factor numerically. Analysis of total equation of state parameter show that, $\omega_{tot} < -1$ and $\omega_{tot} > -1$ imply the phantom-like and quintessence-like behaviors respectively. we have checked the effects of cosmic and viscosity elements on the interaction process. Stability is checked in all the models by the squared velocity of sound.

**Keywords:** Dark energy; Chaplygin gas; Tachyon field; Interaction; Stability.

**Pacs Number:** 95.35.+d; 95.85.-e; 98.80.-k

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# Contents

# 1 Introduction

The most attractive subject in cosmology is the accelerating expansion of the universe which is based on the recent astrophysical data explaining the universe is spatially flat and an invisible cosmic fluid called dark energy with a hugely negative pressure which is responsible for this expansion. This type of matter violates the strong energy condition, i.e.,$\rho + 3p < 0$. There are various phenomenological models describing dark energy. Cosmological constant is the simplest one which gave rise to the $\Lambda$-CDM model, but it suffers from two critical problems; fine tuning and coincidence. The first one relates the small value of cosmological constant and the second problem caused because we live in an epoch that the magnitude of dark energy and dark matter are comparable. Other candidates of dark energy are for examples quintessence [1], phantom [2], quintom [3], tachyon [4], holographic dark energy [5], K-essence [6] and various models of Chaplygin gas. The simplest case of this model based on Chaplygin equation of state [7] to describe the lifting force on a wing of an air plane in aerodynamics. The Chaplygin gas (CG) was not consistent with observational data [8-11]. Therefore, an extension of CG model proposed [12, 13], which is called generalized Chaplygin gas (GCG). However, observational data ruled out such a proposal. Then, GCG extends to the modified Chaplygin gas (MCG) [14]. There is still more extensions such as generalized cosmic Chaplygin gas (GCCG) [15], and modified cosmic Chaplygin gas [16].
On the other hand bulk viscosity plays an important role in the evolution of the universe. The idea that Chaplygin gas may has viscosity first proposed by the Ref. [17] and then developed by [18-21].
In the Ref. [22] a model of varying Chaplyagin gas which interact with a Tachyonic matter considered in framework of general relativity, and investigated the potential $V(\phi)$ and $\omega_{tot}$ numerically. Also, in the Ref. [23] a model of varying generalized Chaplyagin gas which interact with a Tachyonic fluid considered and $V(\phi)$ of the Tachyonic fluid and $\omega_{tot}$ of the mixture investigated graphically.
In this paper we are going to extend Refs. [22] and [23] to complete version of Chaplygin gas and work on the interaction between the mixture of dark matter and dark energy to study their behaviors during evolution of the universe. To achieve this purpose sign changeable interaction is used to include all the eras from the early time until the end. By numerical methods and graphs, the case of varying and the presence of bulk viscosity and cosmic elements are considered. Constants are fixed by imposing the constraint $V \to 0$, and the $\omega_{tot}$ implies the quintessence or phantom behaviors.
We work in the general relativity framework and in FRW metric since has a spatially flat homogeneous and isotropic universe that is suitable for Chaplygin gas as a fluid in it and cause the acceleration expansion of the universe.
This paper organized as the following. In next section we review evolution of Chaplygin gas (CG) to viscous varying modified cosmic Chaplygin gas (VIVAMCCG) and in section 3 we introduce field equations include interaction. In section 4 we study potential $V(\phi)$ of the Tachyonic fluid and $\omega_{tot}$ of the mixture for general case of interaction and various models of scale factor. In section 5 we investigate stability of our model and finally in section 6 we give conclusion.

## 2 Chaplygin gas

In this section we consider perfect fluids as a simple cosmological model and CG [24] is the simplest case to fill the expanding universe. The reason to choose CG is based on the observational data that the equation of state parameter for dark energy can be less than -1. Phantom fields are the same but they have instabilities [25]. The CG equation of state is given by the following [26],

$$p = -\frac{B}{\rho}, \tag{1}$$

where $B$ is a positive constant. The CG is also important in holography [4], string theory [27], and supersymmetry [28]. Equation (1) is connected to string theory and can be achieved by the D-branes Nambu-Goto action which is moving in a (d+2)-dimensional space-time in the light-cone parametrization [29]. CG is the only kind of fluid that accepts a supersymmetric generalization [30]. Another remarkable characteristics of CG is their Euler equations with a large group of symmetry which cause their integrability. CG was studied before [31] in the stabilization of branes [32] and black hole bulks [33]. A negative constant $B$ described in wiggly strings which causes an anti-Chaplygin state equation [34, 35]. CG describes a transition from a decelerated cosmological expansion to the present cosmic acceleration and perhaps submit a deformation of $\Lambda$-CDM models. The inhomogeneous CG can combine dark energy and dark matter and plays the unification role of them [36, 37]. For studying more about CG see Refs. [36-39]. It is also possible to study FRW cosmology of a universe filled with generalized Chaplygin gas (GCG) with the following equation of state [40-42],

$$p = -\frac{B}{\rho^{\alpha}}, \tag{2}$$

with $0 < \alpha \leq 1$. The GCG is also interesting from holography point of view [43]. As we can see the GCG is corresponding to almost dust ($p = 0$) at high density which is not agree completely with our universe. Therefore, modified Chaplygin gas (MCG) with the following equation of state introduced [44, 45].

$$p = \mu\rho - \frac{B}{\rho^{\alpha}}, \tag{3}$$

where $\mu$ is a positive constant. This model is more appropriate choice to have constant negative pressure at low energy density and high pressure at high energy density. The special case of $\mu = \frac{1}{3}$ is the best fitted value to describe evolution of the universe from radiation regime to the $\Lambda$-CDM regime.

The next extension performed by the Ref. [15] where the generalized cosmic Chaplygin gas (GCCG) introduced by the following equation of state,

$$p = -\frac{1}{\rho^{\alpha}}\left[\frac{B}{1+\omega} - 1 + (\rho^{1+\alpha} - \frac{B}{1+\omega} + 1)^{-\omega}\right]. \tag{4}$$

This model can also extend to varying modified cosmic Chaplygin (VAMCCG) gas with the following equation of state,

$$p = \mu\rho - \frac{1}{\rho^{\alpha}}\left[\frac{B(a)}{1+\omega} - 1 + (\rho^{1+\alpha} - \frac{B(a)}{1+\omega} + 1)^{-\omega}\right] \tag{5}$$

where $\omega$ is corresponding cosmic parameter. Here, $B(a)$ is no longer a constant and depend on scale factor via the following relation,

$$B(a) = -\omega(t)B_0 a^{-3(1+\omega(t))(1+\alpha)}, \tag{6}$$

with,

$$\omega(t) = \omega_0 + \omega_1(\frac{t\dot{H}}{H}), \tag{7}$$

It is also possible to include bulk viscosity,

$$p = \mu\rho - \frac{1}{\rho^{\alpha}}\left[\frac{B(a)}{1+\omega} - 1 + (\rho^{1+\alpha} - \frac{B(a)}{1+\omega} + 1)^{-\omega}\right] - 3\varsigma H, \tag{8}$$

where $\varsigma$ is the viscosity coefficient and $H = \dot{a}/\mathrm{a}$ is the Hubble parameter. The equation of state (8) corresponds to viscous varying modified cosmic Chaplygin gas (VIVAMCCG) which is our interesting case in this paper.
The total energy density and pressure relate to the mixture of VIVAMCCG interacting with Tachyon fluid and we define,

$$\rho_{tot} = \rho_{TF} + \rho_{DE}, \tag{9}$$

$$p_{tot} = p_{TF} + p_{DE}, \tag{10}$$

where $\rho_{TF}$ and $p_{TF}$ denote energy density and pressure of Tachyonic fluid respectively. Also, $\rho_{DE}$ and $p_{DE}$ are dark energy density and pressure which given by the equation of state (8).

# 3 Field equations

As we know the Friedmann-Robertson-Walker (FRW) universe in four-dimensional space-time is described by the following metric,

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{11}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and $a(t)$ represents the scale factor. The $\theta$ and $\phi$ parameters are the usual azimuthal and polar angles of spherical coordinates, with $0 \leq \theta \leq \pi$ and $0 \leq \phi < 2\pi$. The coordinates $(t, r, \theta, \phi)$ are called co-moving coordinates. Also, constant $k$ denotes the curvature of the space. The curvature $k$ may be not only positive, corresponding to real finite radius, but also zero or negative, corresponding to infinite or imaginary radius.

The possibilities are called closed ($k = 1$), flat ($k = 0$), and open ($k = -1$). In that case the Einstein equation is given by,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}, \tag{12}$$

where we assumed $c = 1$, $8\pi G = 1$ and $\Lambda = 0$, $k = 0$.
By using the above relations one can obtain the following field equations,

$$H^2 = \frac{\rho_{tot}}{3}, \tag{13}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_{tot} + p_{tot}). \tag{14}$$

The energy-momentum conservation law obtained as the following,

$$\dot{\rho}_{tot} + 3(\rho_{tot} + p_{tot})H = 0, \tag{15}$$

Now, for the interaction between dark energy and matter, splits the equation (15) into the following equations,

$$\dot{\rho}_{DE} + 3(\rho_{DE} + p_{DE})H = Q, \tag{16}$$

where $\rho_{DE}$ is given by the relations (1) to (8) which is $\rho_{VIVAMCCG}$ for our complete model, and,

$$\dot{\rho}_{TF} + 3(\rho_{TF} + p_{TF})H = -Q, \tag{17}$$

where $Q$ is interaction term. Then, the energy density and pressure of a Tachyonic fluid will be,

$$\rho_{TF} = 3H^2 - \rho_{DE}, \tag{18}$$

and

$$p_{TF} = \frac{-Q - \dot{\rho}_{TF}}{3H} - \rho_{TF}, \tag{19}$$

where we used the equation (13) and (17). Then, the equation of state parameter and potential of Tachyonic matter are given by [22, 23],

$$\omega_{TF}(t) = \frac{p_{TF}}{\rho_{TF}}, \tag{20}$$

$$V(\phi) = \sqrt{-\rho_{TF}p_{TF}}, \tag{21}$$

and $\omega_{tot}$ is the equation of state parameter of the mixture which given by,

$$\omega_{tot} = \frac{p_{TF} + p_{DE}}{\rho_{TF} + \rho_{DE}}. \tag{22}$$

In order to investigate potential (21) and the total equation of state parameter (22) we use conservation equation (16) to obtain $\rho_{DE}$. Then, we use the equation (13) to obtain $\rho_{TF}$. Therefore, the equation (14) gives us $p_{TF}$, and hence $V(\phi)$ and $\omega_{tot}$ will be obtained.

# 4 Sign changeable interaction and scale factors

One of the ways to solve the cosmological coincidence problem is to consider the interaction between the components on phenomenological level. Generally, interaction could be considered as a function of energy densities and their derivatives: $Q(\rho_i, \dot{\rho}_i, ...)$. Interactions considered as: $Q = 3Hb\rho_m$, where $b > 0$ is a coupling constant. The general form is $Q = 3Hb\gamma\rho_i + \gamma\dot{\rho}_i$, where $i = TF, DE, tot$, and $\gamma$ is dimensionless constant [46].These kind of interactions are either positive or negative and cannot change sign. A sign-changeable interaction is of the following form,

$$Q = q(\gamma\rho + 3bH\rho), \tag{23}$$

where $q$ is the deceleration parameter which is given by,

$$q = -(1 + \frac{\dot{H}}{H^2}). \tag{24}$$

Now, by putting the sign-changeable interaction and the equation of state for viscous varying modified cosmic Chaplygin gas (8) in the equation (16), we get to a differential equation as the following,

$$\dot{\rho} + 3\left[\rho + \mu\rho - \frac{1}{\rho^\alpha}\left[\frac{B(a)}{1+\omega} - 1 + (\rho^{1+\alpha} - \frac{B(a)}{1+\omega} + 1)^{-\omega}\right] - 3\varsigma H\right] H = q(\gamma\rho + 3bH\rho), \tag{25}$$

which can be solved numerically for the $\rho$ during the time. Then $p_{TF}$ and $\rho_{TF}$ can be achieved by equations (18) and (19). Here, the variables are fixed to $V_{TF} \to 0$.
According to the accelerating expansion of the universe, we work on three kinds of scenarios which are based on different eras in the evolutionary process of the universe which all of them consist of a kind of expanding exponential scale factor as the followings.

## 4.1 Emergent scenario

The universe in the emergent scenario has some interesting properties as the following. The universe at large scale is isotropic and homogeneous and there is no time-like singularity. The universe is accelerating and may contains exotic matter such as CG. The scale factor in this scenario is given by,

$$a(t) = a_0(B + e^{Kt})^m, \tag{26}$$

where $a_0 > 0$, $K > 0$, $B > 0$, and $m > 1$ [36].
First, we consider the simplest case of modified cosmic Chaplygin gas interacting with Tachyon fluid in emergent era and reconstruct the tachyon potential and total equation of state parameter numerically (Fig. 1).
By imposing the bulk viscous parameter $\varsigma \neq 0$ and variable parameter $B(a)$ in the equation of state, we have the VIVAMCCG, which is the general form. Here we suppose various models and find their behaviors in respect to their $\omega_{tot}$ and according to the constraint $V_{TF} \to 0$.

Analysis of Tachyon potential $V(\phi)$ and $\omega_{tot}$ shows that in all models for the emergent scenario and during whole evolution of the universe from the beginning to the end, the total equation of state parameter is $\omega_{tot} < -1$ which means the phantom-like behavior (Figs. 2-7).This shows that the Emergent scenario is useful for describing the universe because it is in agreement with the real data. In the presence of cosmic element in the VIVAMCCG in comparison to the VIVAMCG, the Tachyon potential has a faster decreasing in time. In VIMCCG and VAMCCG in comparison with the case of MCCG, The presence of viscosity and varying condition cause slower vanishing for the potential.

### 4.2 Intermediate scenario

For the scale factor corresponding to intermediate scenario we have,

$$a(t) = e^{\lambda t^{\beta}}, \tag{27}$$

where $\lambda > 0$ and $0 < \beta < 1$ [37, 38].
Numerical research on the Tachyon potential and total equation of state parameter in this era with the intermediate scale factor reveals the quintessence-like behavior which is based on the $\omega_{tot} > -1$ (Figs. 8-13).
In this case, the Tachyon potential of the VIVAMCCG vanishes slower than the VIVAMCG. It the VIMCG model, it takes the longest time to $V_{TF} \to 0$. Here, we found a different reaction and $V_{TF}$ in the VIVAMCCG decreases slower by the cosmic effect.

### 4.3 Logamediate scenario

The Logamediate scenario of the universe is motivated by considering a class of possible cosmological solutions with indefinite expansion. In this model the scale factor showing the accelerating expansion of the universe is given by,

$$a(t) = e^{x(\ln(t))^{\beta}}, \tag{28}$$

where $x > 0$, and $\beta > 1$ [37].
The same process for the Logamediate scale factor gives the Figs. 14-18 which are similar to the Intermediate case consistent with the quintessence-like behavior. Here, VIVAMCCG, acts has a rather slower decrease in the value of $V_{TF}$ than the VIVAMCG potential.

## 5 Stability

Now, we are interested to check the stability to find the best model for explaining the accelerating expansion of the universe. Positive and bounded squared sound velocity of Chaplygin gas is a remarkable feature that is a non-trivial fact of fluids based on their negative pressure

$$\upsilon_s^2 = \frac{\partial P}{\partial \rho}. \tag{29}$$

Here, we study the stability of these variety of models with respect to the positive quantity for $v_s^2$. This is important to mention that the parameters are fixed according to $V_{TF} \to 0$ in the interaction.

### 5.1 Emergent scenario

In this case, the models VAMCCG, VAMCG, VIMCG and VIVAMCG are stable during the whole time and the model of MCCG has no stability here. For the model of VIMCCG, there is no stability until $t = 15$ and for VIVAMCCG, as the general form, the stability is present from the beginning but after the time $t = 108$, no stability exits in the model.

### 5.2 Intermediate scenario

In the general model of VIVAMCCG the stability stays only till early moments $t = 9$ and stability continues no more. That is like the emergent scenario but with this difference here that the stability vanishes so soon in comparison with the similar model in emergent kind. The models VIMCG, VAMCG, VIMCCG and VIVAMCG are stable all the time of evolution. In the VAMCCG the stability disappears very soon at $t = 3$ that is a distinct situation of this scenario.

### 5.3 Logamediate scenario

For VIVAMCCG in logamediate scenario, the stability holds in early time before $t = 18$ and exactly like the other scenarios during the late time the model misses stability. In the other models as VIVAMCG, VAMCCG and VAMCG the stability is always available during the whole evolution of the universe in respect to the time. But in the VIMCCG as the emergent similar case, the stability holds until the time $t = 432$.

## 6 Conclusion

In this paper we extend Ref. [22] and [23] to the case of viscous varying modified cosmic Chaplygin gas and studied interaction with Tachyonic fluid. We considered general form of interaction and three different cases of the universe which are Emergent, Intermediate and Logamediate scenarios. In the Emergent case, we found the phantom-like behavior due to the $\omega_{tot}$ and quintessence-like behaviors in the Intermediate and Logamediate era is obvious. The comparison between the values of $\omega_{tot}$ in the case of $MCCG$ and in the presence of viscosity shows a tiny difference that is negligible. Also, The cosmic element causes no comparable difference in the $\omega_{tot}$. By taking $\varsigma = 0.3$ the values of $V_{TF}$ and $\omega_{tot}$ decreases a little.
The viscosity causes a little delay in the vanishing procedure of the Tachyon potential and the cosmic element appears by the same feature.
Stability of the general model of VIVAMCCG of all the three scenarios are contemporary during the time and vanishes in the late time of expanding evolution of the universe. It

shows that the viscous varying modified cosmic Chaplygin gas model is valid for the early universe.
Since the Phantom-like behavior is more consistent with the observational data, Then the Emergent scenario is better than the Logamediate and Intermediate ones and the stable models in this scenario are the best ones among all the cases we have considered here. These suitable models are VAMCCG, VAMCG, VIMCG and VIVAMCG in Emergent scenario.

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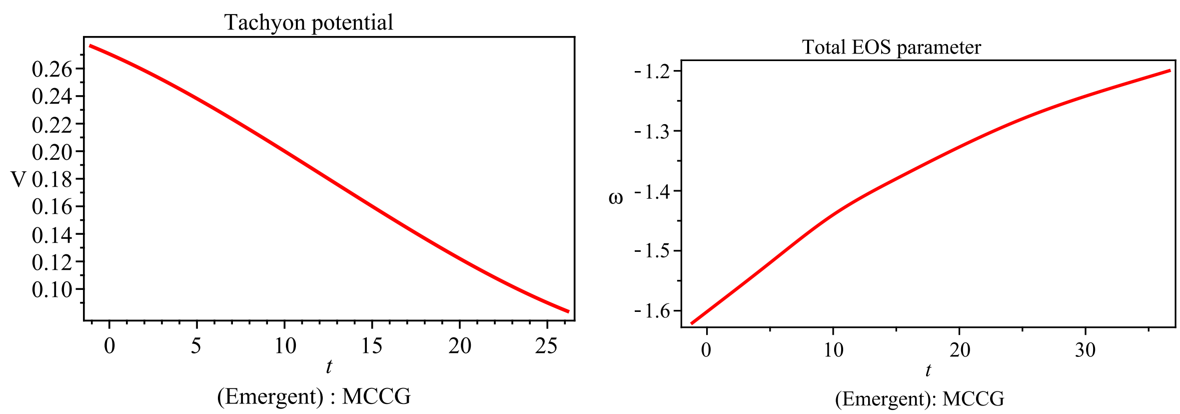



Figure 1: Plot of $V$ and $\omega_{tot}$ in terms of time with $m = 1.1$, $A = -0.03$, $b = 1$, $\gamma = 0.7$, $B = 1$, $a_0 = 0.6$, $\mu = 0.3$, $\omega = -0.5$, $\alpha = 0.5$ and $K = 0.03$.

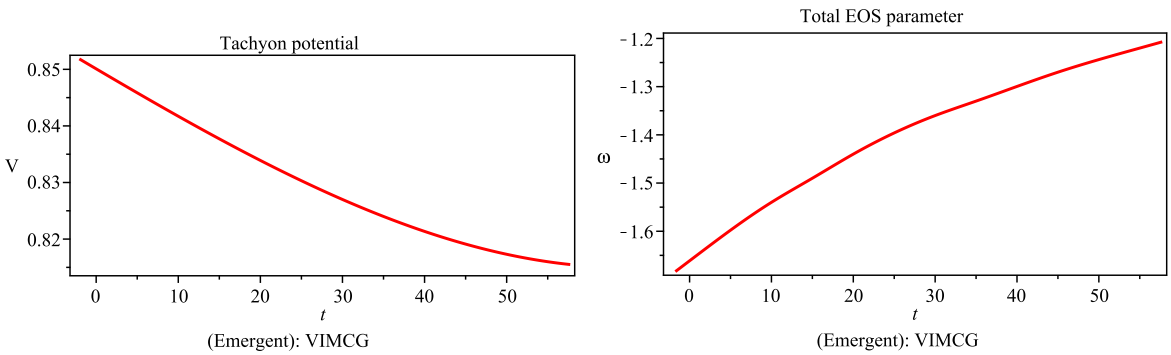



Figure 2: Plot of $V$ and $\omega_{tot}$ in terms of time with $m = 1$, $A = 1$, $b = 1$, $\gamma = 1$, $B = 1$, $a_0 = 1$, $\mu = 0.3$, $\varsigma = 1$, $\alpha = 0.5$ and $K = 0.02$.

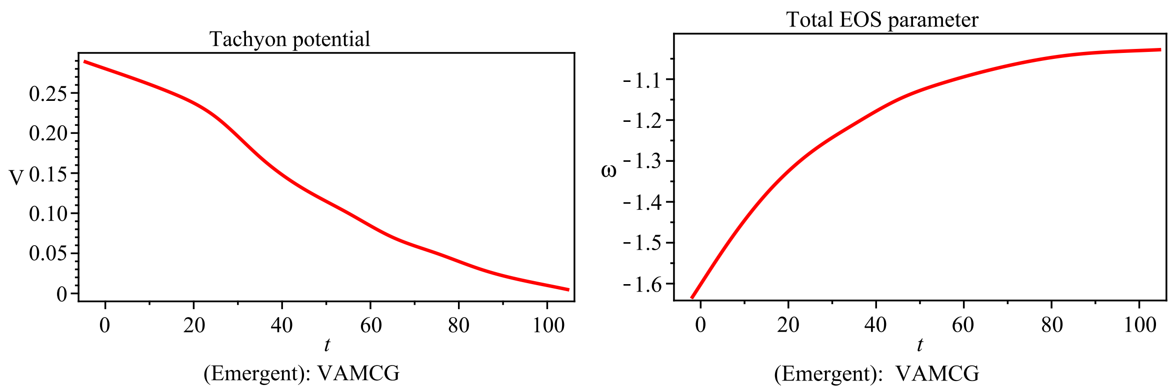



Figure 3: Plot of $V$ and $\omega_{tot}$ in terms of time with $m = 1.1$, $b = 0.1$, $\gamma = 0.7$, $B = 1$, $a_0 = 0.6$, $i = -0.6$, $j = -0.08$, $\mu = 0.3$, $\alpha = 0.5$ and $K = 0.03$.

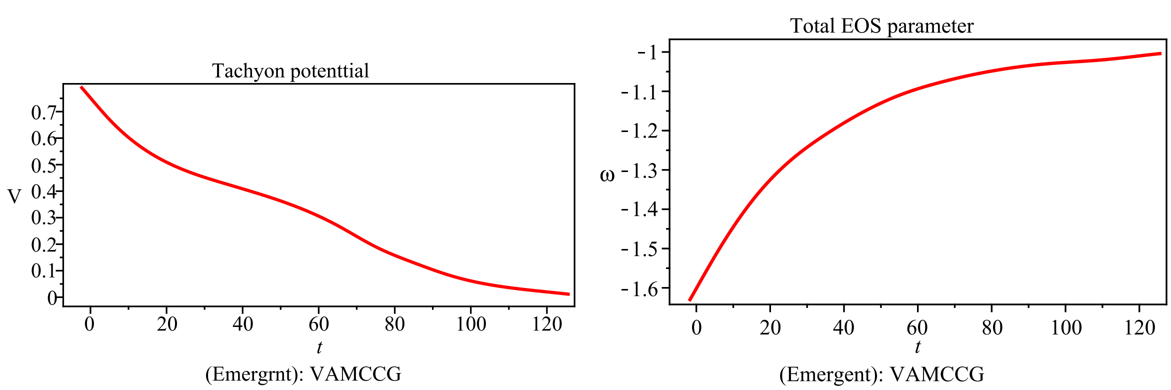



Figure 4: Plot of $V$ and $\omega_{tot}$ in terms of time with $m = 1.1$, $b = 1$, $K = 0.03$, $\gamma = 0.7$, $B = 1$, $a_0 = 0.6$, $i = -0.6$, $j = -0.8$, $\mu = 0.3$, $\omega = -0.5$ $\alpha = 0.5$ and $K = 0.03$.

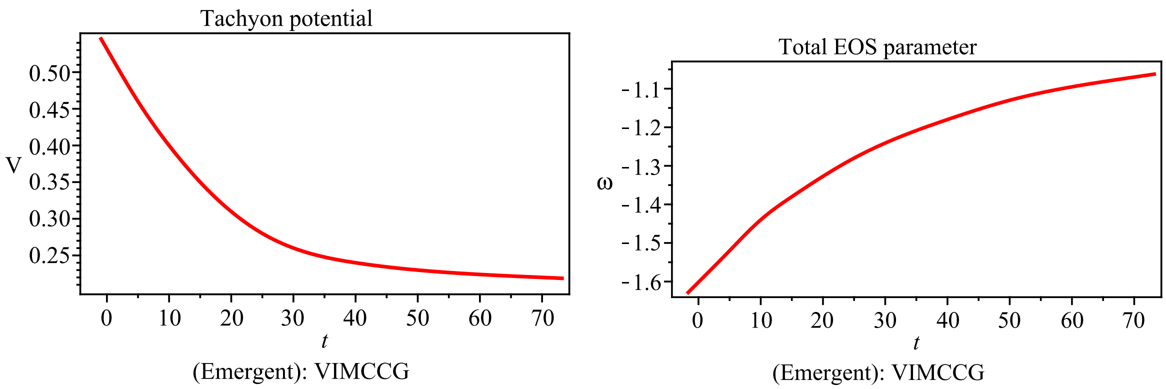



Figure 5: Plot of $V$ and $\omega_{tot}$ in terms of time with $m = 1.1$, $A = 0.1$, $b = 1$, $K = 0.03$, $\gamma = 0.7$, $B = 1$, $a_0 = 0.6$, $\mu = 0.3$, $\varsigma = 1$, $\omega = -0.5$, $\alpha = 0.5$ and $K = 0.03$.

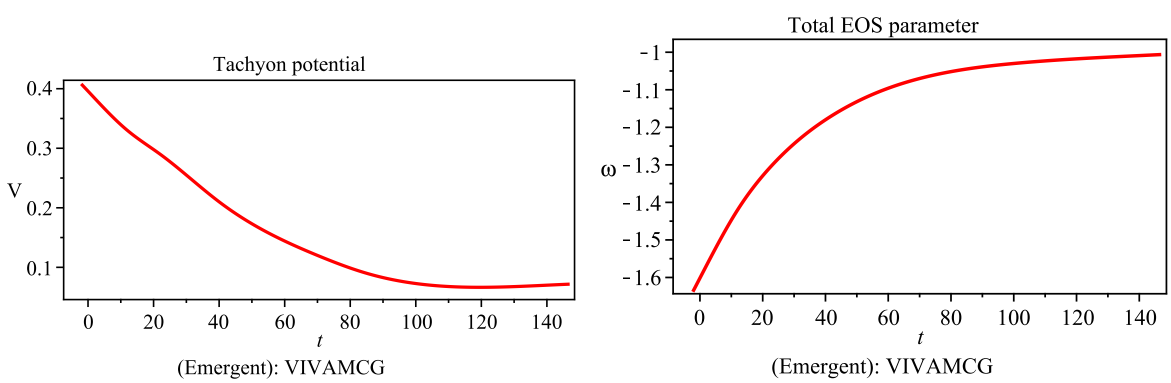



Figure 6: Plot of $V$ and $\omega_{tot}$ in terms of time with $m = 1.1$, $b = 0.1$, $K = 0.03$, $\gamma = 0.7$, $B = 1$, $a_0 = 0.6$, $i = -0.6$, $j = -0.08$, $\mu = 0.3$, $\varsigma = 1$, $\alpha = 0.5$ and $K = 0.03$.

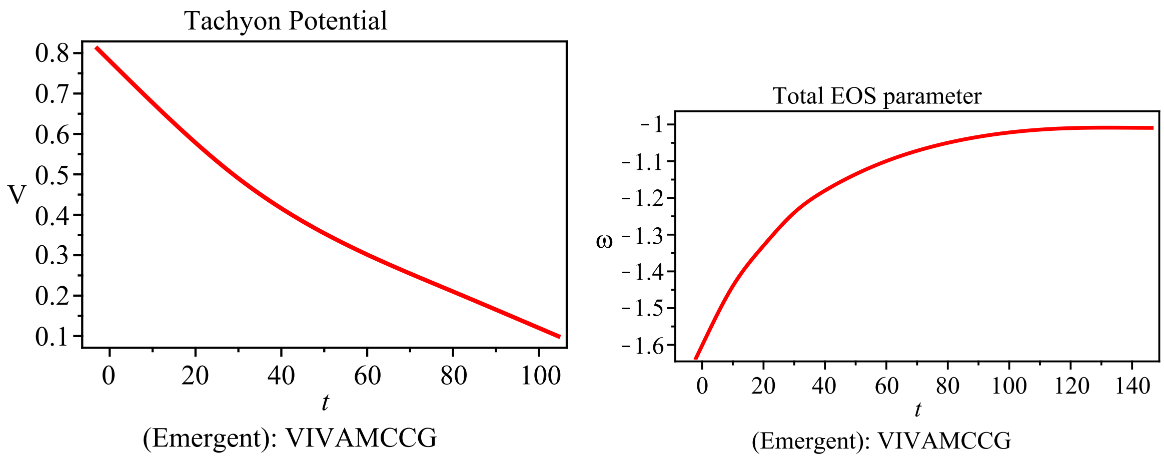



Figure 7: Plot of $V$ and $\omega_{tot}$ in terms of time with $m = 1.1$, $b = 1$, $K = 0.03$, $\gamma = 0.7$, $B = 1$, $a_0 = 0.6$, $i = -0.6$, $j = -0.8$, $\mu = 0.3$, $\varsigma = 1$, $\omega = -0.5$ $\alpha = 0.5$ and $K = 0.03$.

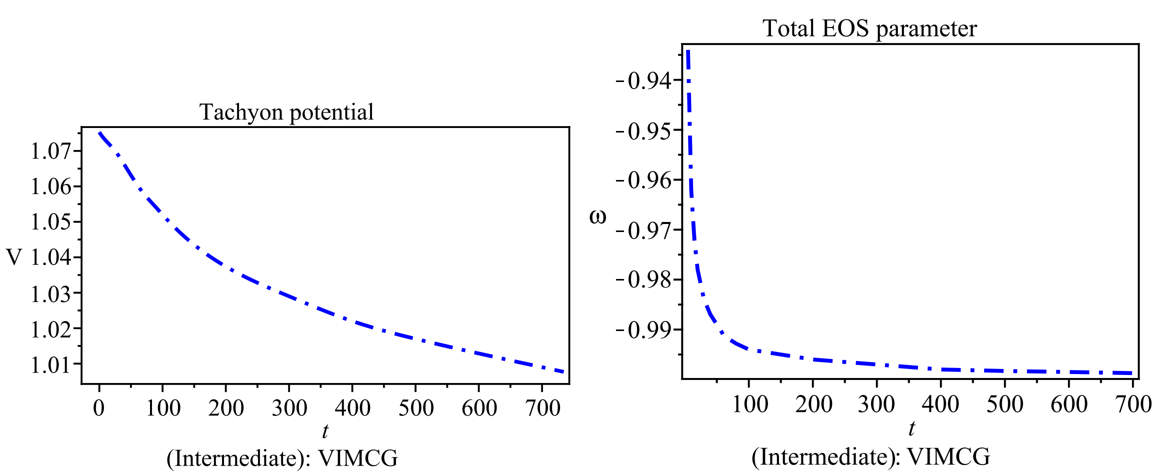



Figure 8: Plot of $V$ and $\omega_{tot}$ in terms of time with $\mu = 0.3$, $\gamma = 0.7$, $\lambda = 0.7$, $\beta = 0.8$, $\varsigma = 1$, $A = 1$, $b = 0.1$, and $\alpha = 0.5$.

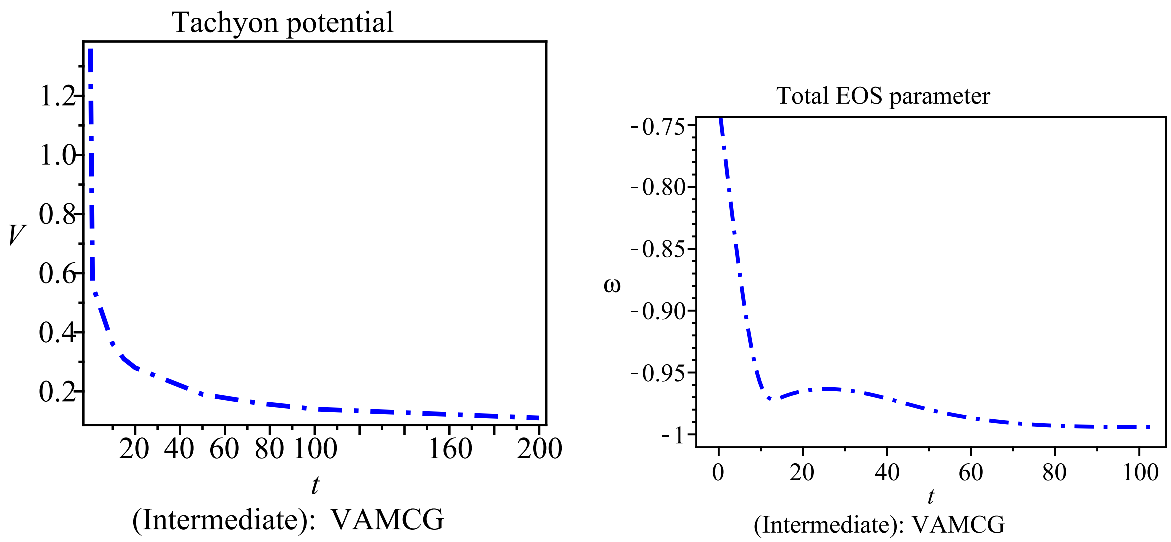



Figure 9: Plot of $V$ and $\omega_{tot}$ in terms of time with $\mu = 0.3$, $\gamma = 0.7$, $\lambda = 0.7$, $\beta = 0.8$, $b = 0.1$, $i = -0.6$, $j = -0.03$ and $\alpha = 0.5$.

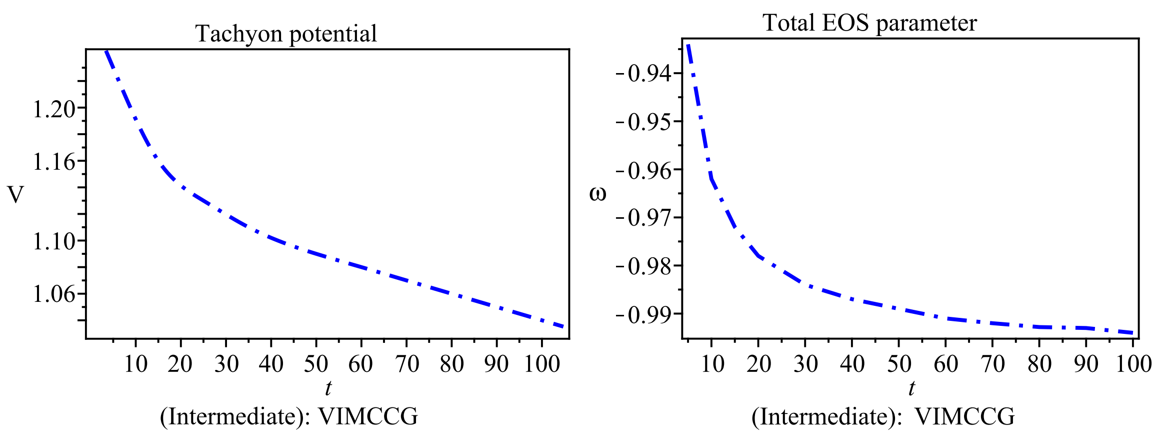



Figure 10: Plot of $V$ and $\omega_{tot}$ in terms of time with $\mu = 0.3$, $\gamma = 0.7$, $\lambda = 0.7$, $\beta = 0.8$, $\varsigma = 1$, $A = 0.4$, $b = 0.1$, $\omega = -0.5$ and $\alpha = 0.5$

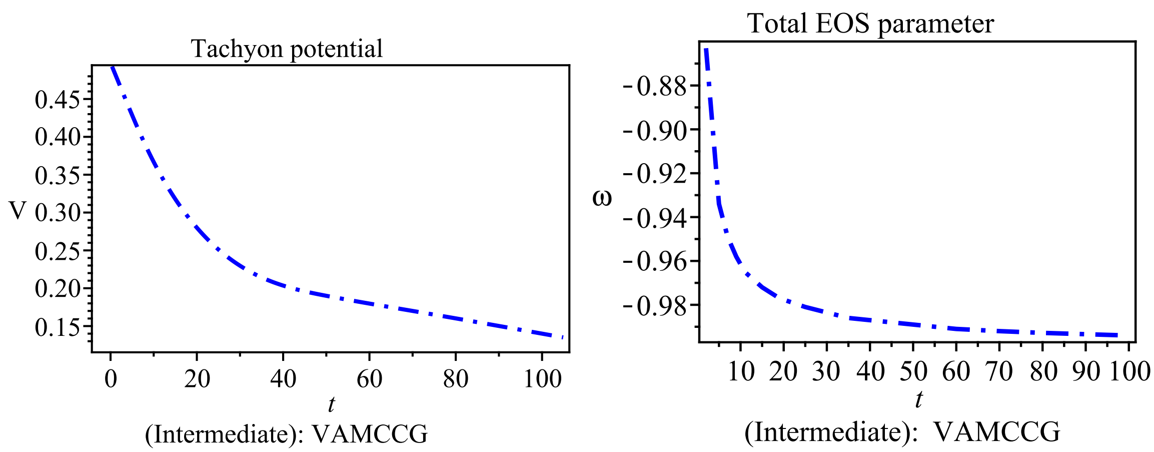



Figure 11: Plot of $V$ and $\omega_{tot}$ in terms of time with $\mu = 0.3$, $\gamma = 0.7$, $\lambda = 0.7$, $\beta = 0.8$, $\varsigma = 1$, $i = -0.6$, $j = -0.03$, $b = 0.1$, $\omega = -0.5$, and $\alpha = 0.5$.

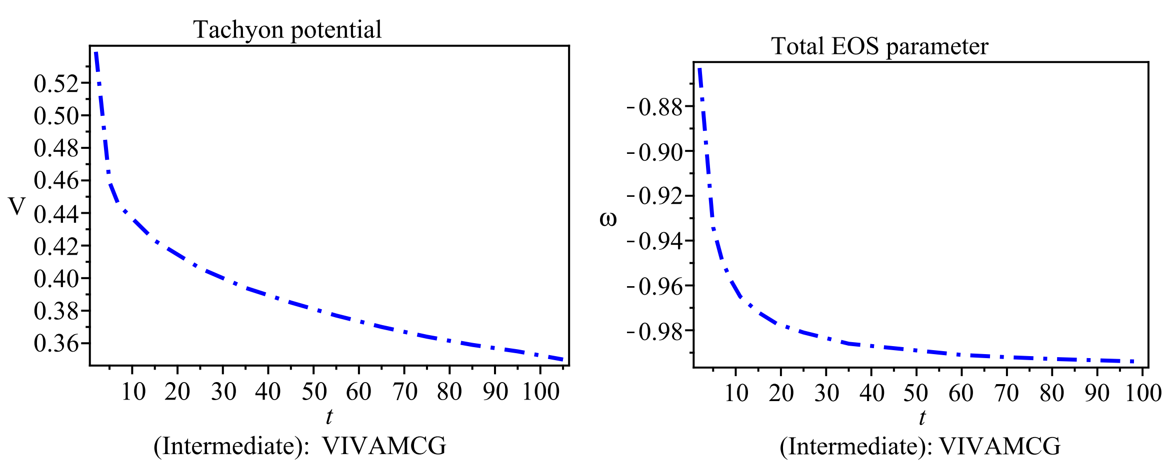



Figure 12: Plot of $V$ and $\omega_{tot}$ in terms of time with $\mu = 0.3$, $\gamma = 0.7$, $\lambda = 0.7$, $\beta = 0.8$, $\varsigma = 1$, $i = -0.6$, $j = -0.03$, $b = 0.1$, and $\alpha = 0.5$.

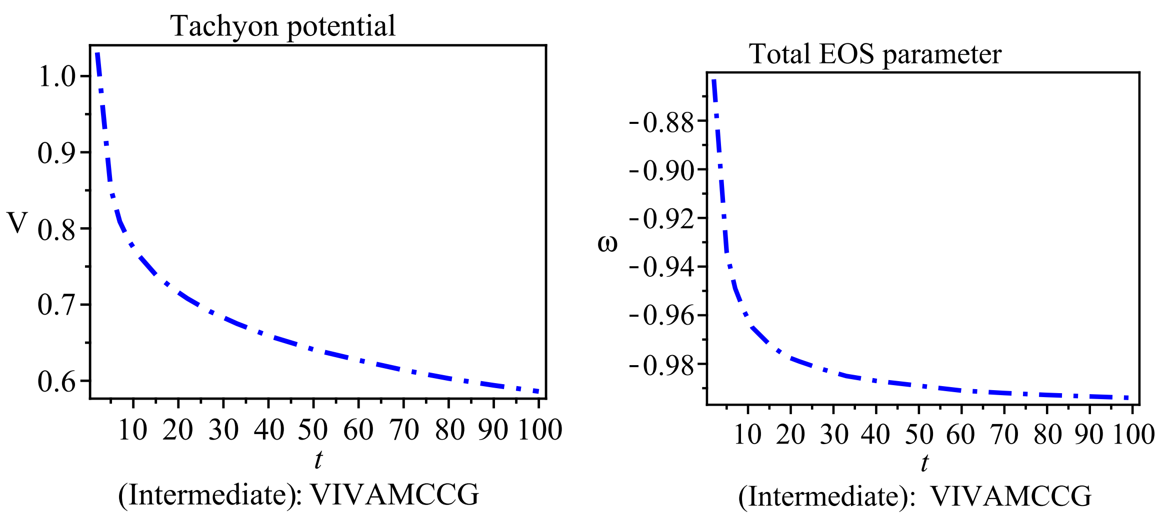



Figure 13: Plot of $V$ and $\omega_{tot}$ in terms of time with $\mu = 0.3$, $\gamma = 0.7$, $\lambda = 0.7$, $\beta = 0.8$, $\varsigma = 1$, $i = -0.6$, $j = -0.03$, $b = 0.1$, $\omega = -0.5$, and $\alpha = 0.5$.

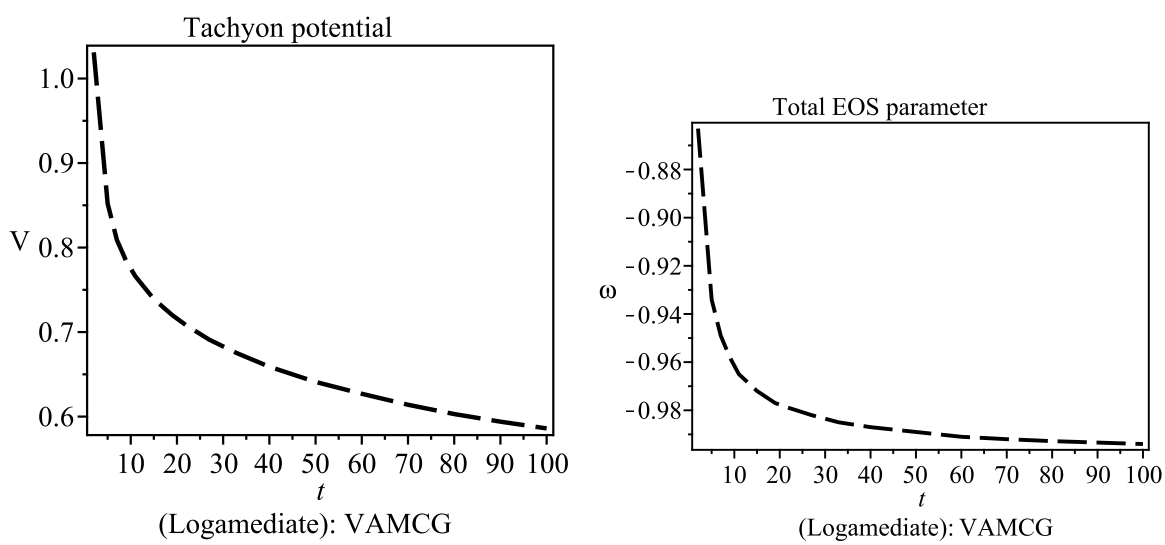
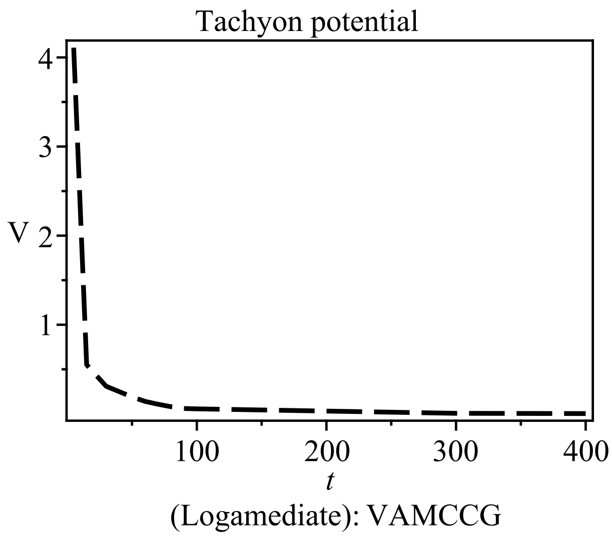



Figure 14: Plot of $V$ and $\omega_{tot}$ in terms of time with $x = 0.05$, $b = 0.4$, $\beta = 2$, $\gamma = 0.1$, $\alpha = 0.5$, $\mu = 0.3$, $i = -0.8$ and $j = -0.4$.



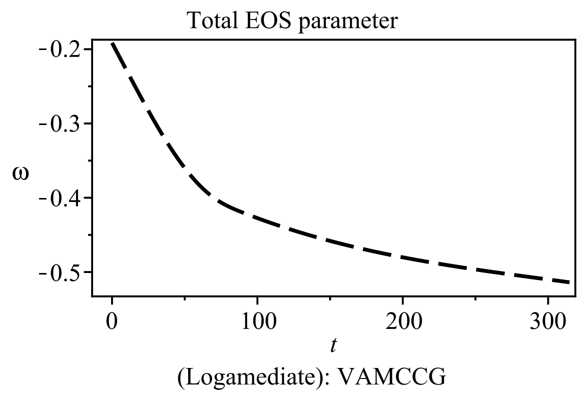
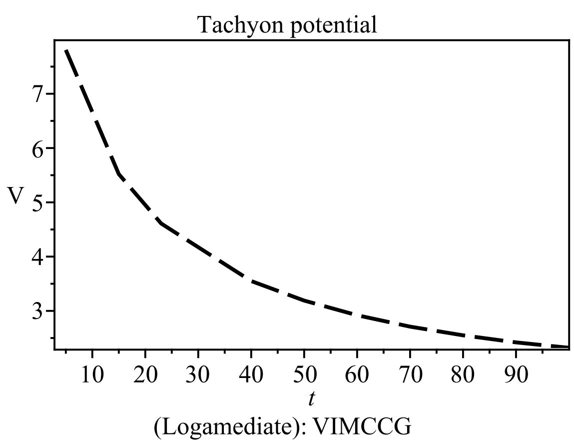




Figure 15: Plot of $V$ and $\omega_{tot}$ in terms of time with $x = 0.1$, $b = 0.4$, $\beta = 2$, $\gamma = 0.1$, $\alpha = 0.5$, $\mu = 0.3$, $\omega = -0.5$, $i = -0.8$ and$j = -0.4$.



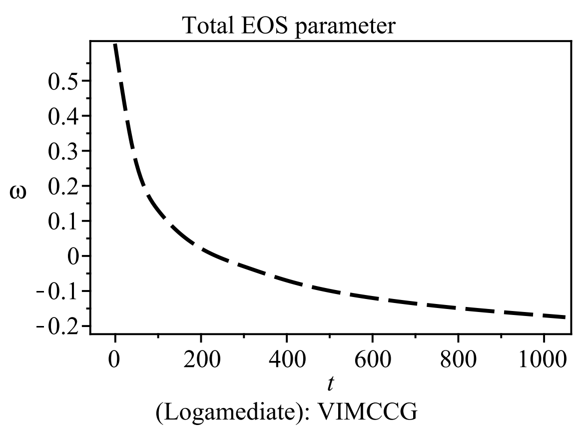




Figure 16: Plot of $V$ and $\omega_{tot}$ in terms of time with $x = 0.05$, $b = 0.4$, $\beta = 2$, $\gamma = 0.1$, $\alpha = 0.5$, $\mu = 0.3$, $\varsigma = 1$,$\omega = -0.5$ and $A = 1$.

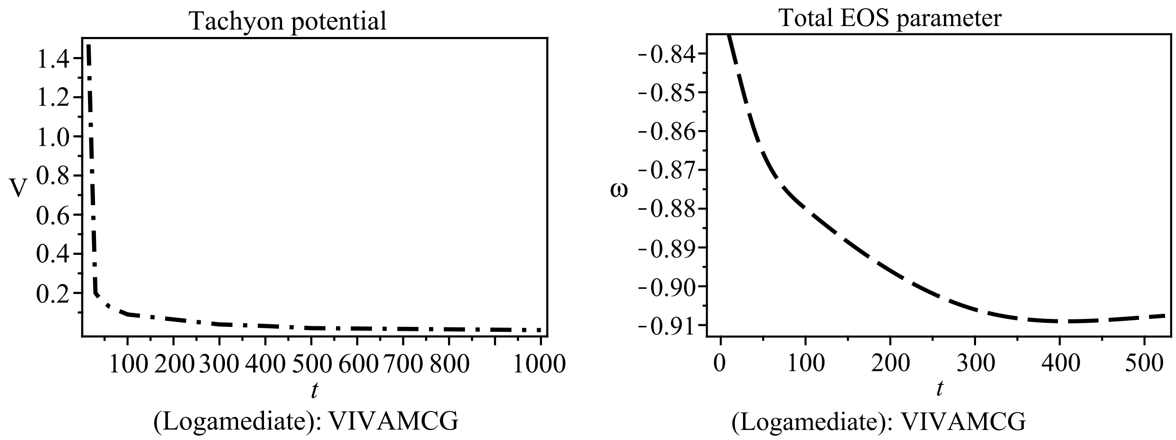



Figure 17: Plot of $V$ and $\omega_{tot}$ in terms of time with $x = 0.5$, $b = 0.4$, $\beta = 2$, $\gamma = 0.1$, $\alpha = 0.5$, $\mu = 0.3$, $\varsigma = 1$, $i = -0.8$ and $j = -0.7$.

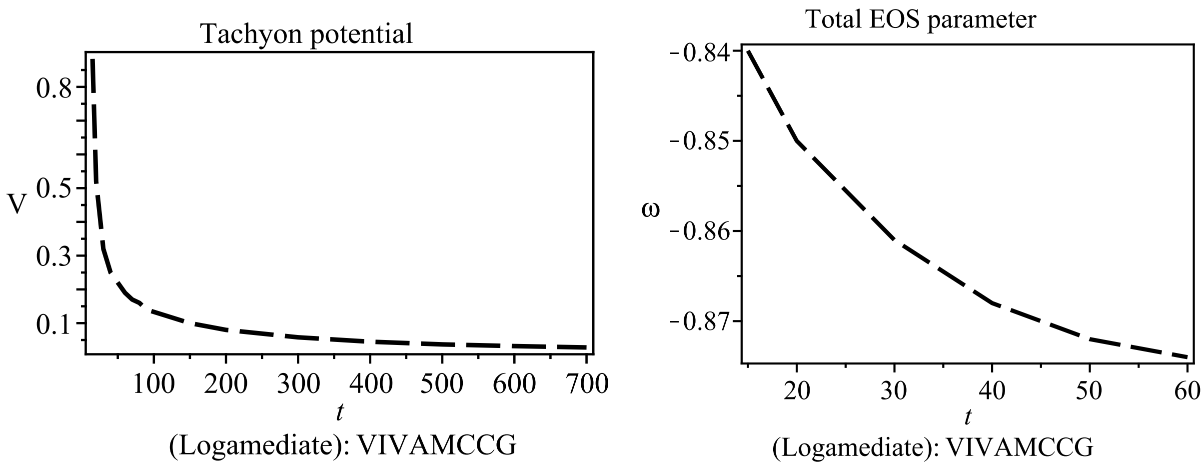



Figure 18: Plot of $V$ and $\omega_{tot}$ in terms of time with $x = 0.5$, $b = 0.4$, $\beta = 2$, $\gamma = 0.1$, $\alpha = 0.5$, $\mu = 0.3$, $\varsigma = 1$, $i = -0.8$, $j = -0.7$ and $\omega = -0.5$.